Quarkonium Production at Hadron-Hadron Colliders

Geoffrey Bodwin
Argonne National Lab

- Factorization of the Inclusive Production Cross Section
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- Comparisons of NRQCD Factorization with Hadron-Hadron Experiments
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- Summary

Factorization of the Inclusive Quarkonium Production Cross Section

- In heavy-quarkonium hard-scattering production, high-momentum scales appear: m and p_T.
- We would like to use NRQCD to separate the perturbative physics at these high-momentum scales from the low-momentum, nonperturbative effects in the heavy-quarkonium dynamics.
- The probability for a $Q\bar{Q}$ pair to evolve into a heavy quarkonium can be calculated as a vacuum-matrix element in NRQCD:

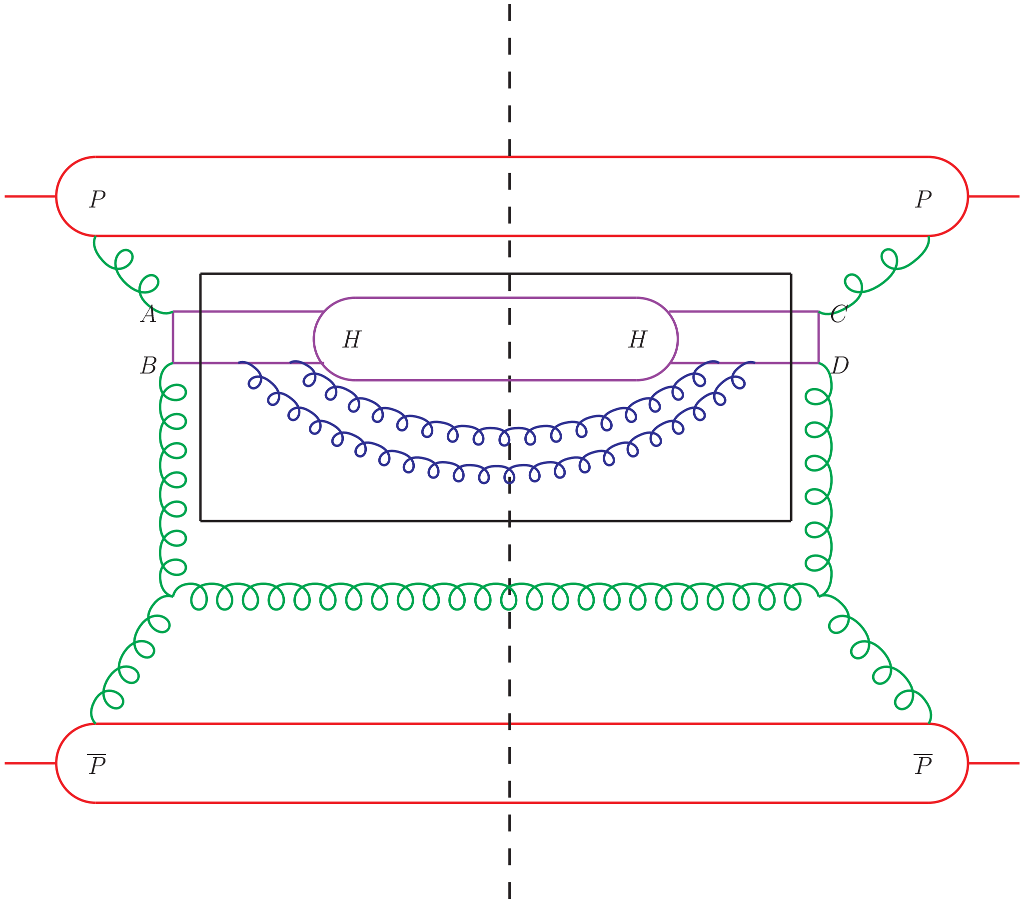
$$\mathcal{O}_n^H(\Lambda) = \langle 0|\chi^\dagger \kappa_n \psi \left(\sum_X |H + X\rangle\langle H + X| \right) \psi^\dagger \kappa_n' \chi |0\rangle .$$

- This is the matrix element of a four-fermion operator, but with a projection onto an intermediate state of the quarkonium H plus anything.
  - κ_n and κ_n' are combinations of Pauli and Color matrices.

- Conjecture (GTB, Braaten, Lepage (1995)):
  The inclusive cross section for producing a quarkonium at large momentum transfer (p_T) can be written as a sum of "short-distance" coefficients times NRQCD matrix elements.

$$\sigma(H) = \sum_n F_n(\Lambda)\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle .$$

- The part of the diagram inside the box corresponds to an NRQCD matrix element.



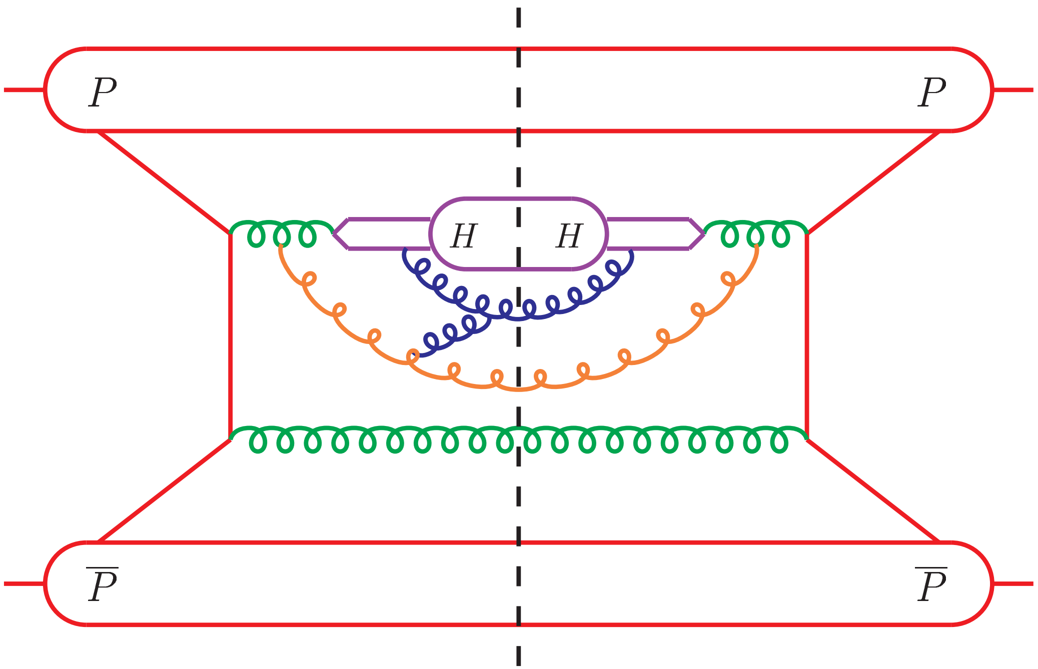
- The points $A(C)$ and $B(D)$ are within $\sim 1/m$ of each other.
  - Kinematics implies that the virtual Q is off shell by order m.
- The points $A(B)$ and $C(D)$ are within $1/p_T$ of each other.
  - The part of the diagram outside the box is insensitive to changes of momentum flow from $A(B)$ to $C(D)$ of order p_T.

- The “short-distance” coefficients $F_n(\Lambda)$ are essentially the process-dependent partonic cross sections to make a $Q\bar{Q}$ pair convolved with the parton distributions.
  - They have an expansion in powers of α_s.
- The operator matrix elements are universal (process independent).
  - Only the color-singlet production and decay matrix elements are simply related.
- The matrix elements have a known scaling with v.
- The NRQCD factorization formula is a double expansion in powers of α_s and v.
- A key feature of NRQCD factorization:
  Quarkonium production can occur through color-octet, as well as color-singlet, $Q\bar{Q}$ states.
- If we drop all of the color-octet contributions and retain only the leading color-singlet contribution, then we have the color-singlet model (CSM).
  - Inconsistent for P-wave production: IR divergent.

Status of a Proof of Factorization

- A proof is complicated because gluons can dress the basic production process in ways that apparently violate factorization.
- A proof of factorization would involve a demonstration that diagrams in each order in α_s can be re-organized so that
  - All soft singularities cancel or can be absorbed into NRQCD matrix elements,
  - All collinear singularities and spectator interactions can be absorbed into parton distributions.
- Nayak, Qiu, Sterman (2005, 2006): The color-octet NRQCD matrix elements must be modified by the inclusion of eikonal lines to make them gauge invariant.
  - The eikonal lines are path integrals of the gauge field running from the creation and annihilation points to infinity.
  - Essential at two-loop order to allow certain soft contributions to be absorbed into the matrix elements.
  - Does not affect existing phenomenology, which is at tree order or one-loop order in the color-octet contributions.

- Nayak, Qiu, Sterman (2005, 2006): A key difficulty in proving factorization to all orders is the treatment of gluons with momenta of order m in the quarkonium rest frame.



- If the orange gluon has momentum of order m, it can't be absorbed into the NRQCD matrix element as a quarkonium constituent.
- But the orange gluon can have non-vanishing soft exchanges with the quarkonium constituents.
- The orange gluon can be treated as the eikonal-line part of the NRQCD matrix element, provided that the answer does not depend on the direction of the eikonal line (universality of the matrix elements).

- Nayak, Qiu, Sterman (2005, 2006): At two-loop order, the eikonal lines contribute but a "miracle" occurs: The dependence on the direction of the eikonal line cancels.
- In general, factorization of the inclusive cross section beyond two-loop order is still an open question.
- An all-orders proof is essential because the α_s associated with soft gluons is not small.

- Nayak, Qiu, Sterman (2007, 2008): If an additional heavy quark is approximately co-moving with the $Q\bar{Q}$ pair that forms the quarkonium, there are soft color exchanges between the heavy quark and the $Q\bar{Q}$ pair.
  - This process does not fit into the NRQCD factorization picture.
    It requires production matrix elements that contain additional heavy quarks beyond the $Q\bar{Q}$ pair.
  - The process is nonperturbative: It can't be calculated reliably.
  - Can search for the process experimentally:
    The signature is additional heavy-meson production in a narrow cone ($\sim\ mv/p_T$) around the quarkonium.
  - This effect might be eliminated from the measured cross section through the use of an isolation cut.

The Fragmentation Approach

Kang, Qiu, and Sterman (2010)

- Writes the cross section in terms of
  - single-parton production cross sections convolved with the fragmentation functions for a single parton into a quarkonium

$$d\hat{\sigma}_{A+B\to i+X} \otimes D_{i\to H}$$

  - $Q\bar{Q}$ production cross sections convolved with fragmentation functions for a $Q\bar{Q}$ pair into a quarkonium

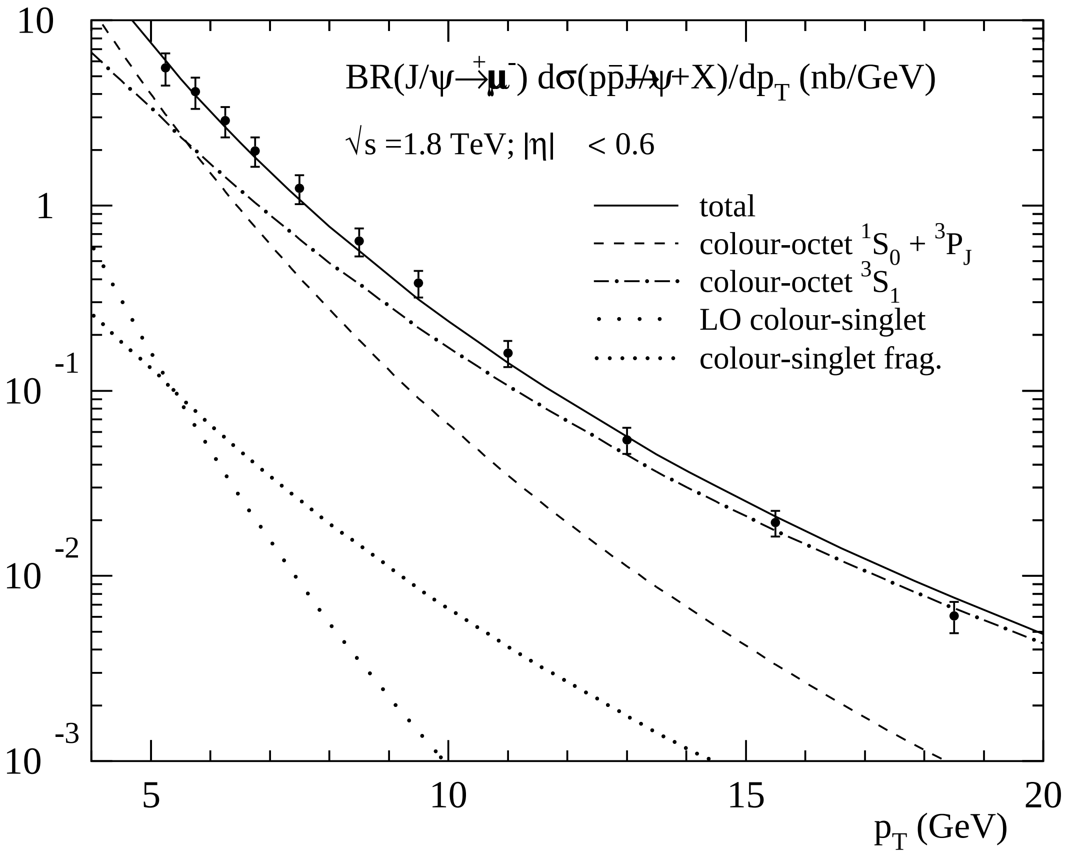
$$d\hat{\sigma}_{A+B\to Q\bar{Q}+X} \otimes D_{Q\bar{Q}\to H}$$

- Re-organizes the perturbation expansion as an expansion in powers of $1/p_T$.
- Believed to hold to all orders in perturbation theory up to corrections of order m_Q^4/p_T^4.
- If NRQCD factorization holds, then the fragmentation functions can be written as a sum of NRQCD matrix elements times perturbatively calculable short-distance coefficients.
- See Qiu's talk at QWG2010 (conferences.fnal.gov/QWG2010) and Eur. Phys. J. C **71**, 1534 (2011).

Comparisons of NRQCD Factorization with Experiment

Quarkonium Production and Polarization at the Tevatron

Production Cross Section in LO



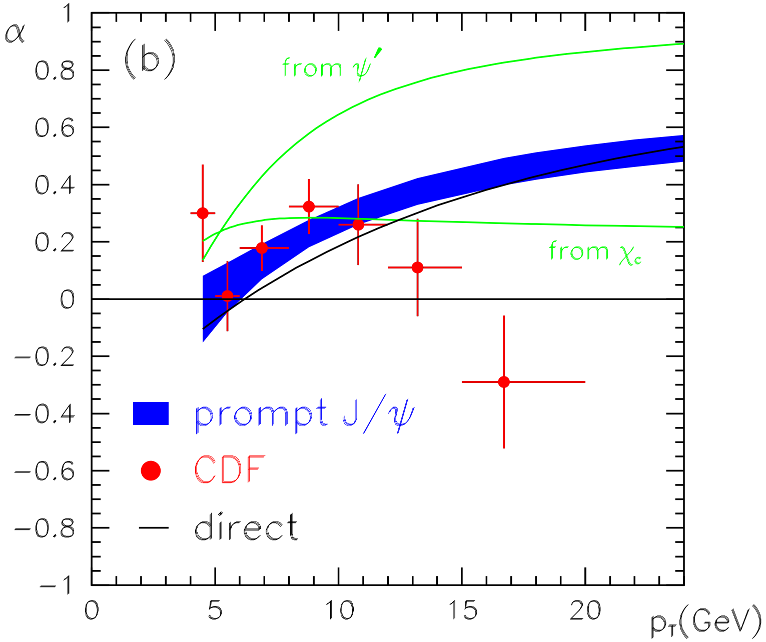
- The CDF (1997) data are more than an order of magnitude larger than the LO predictions of the color-singlet model.
- p_T distributions are consistent with NRQCD prediction (Krämer (2001)), but not with the LO color-singlet model.
- Color-octet matrix elements are determined from fits to the data.
- Good fits for J/ψ, $\psi(2S)$, χ_c, $\Upsilon(1S)$ production, as well.
- Use color-octet matrix elements from these fits to predict quarkonium production in other processes (test universality).

Polarization in LO

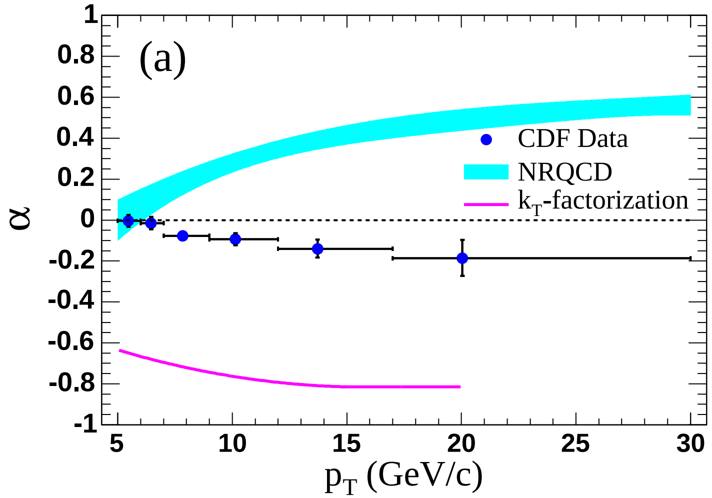
- Transverse quarkonium polarization may be a signature of the color-octet mechanism.
- In LO quarkonium production at large p_T ($p_T \gtrsim 4m_c$ for J/ψ), gluon fragmentation via the color-octet ${}^3S_1$ $Q\bar{Q}$ state dominates.
- At large p_T, the gluon is nearly on mass shell, and, so, is transversely polarized.
- In color-octet gluon fragmentation, most of the gluon's polarization is transferred to the quarkonium (Cho, Wise (1994)).
  - Spin-flip interactions are suppressed as v^2.
  - Verified in a lattice calculation of decay matrix elements (GTB, Lee, Sinclair (2005)).
- Radiative corrections dilute this (Beneke, Rothstein (1995); Beneke, Krämer (1996)).

J/ψ Polarization in LO

Run I:



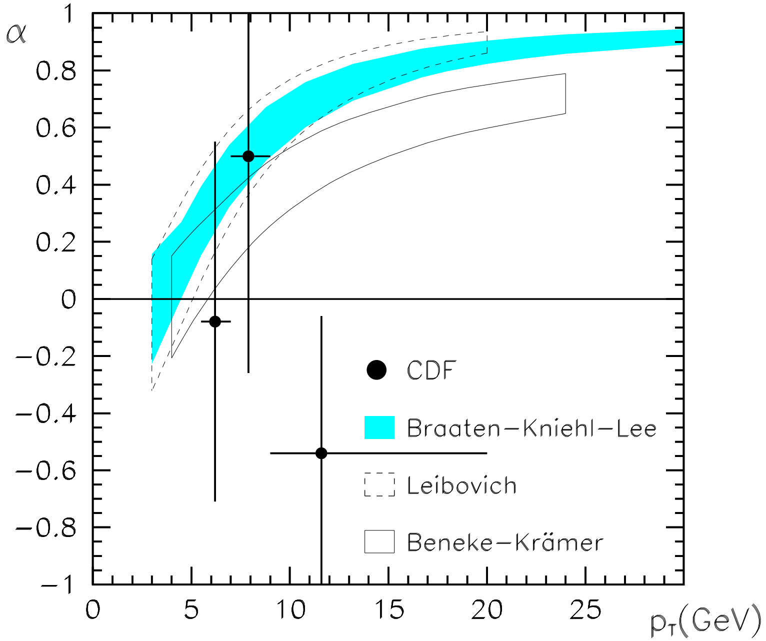
Run II:



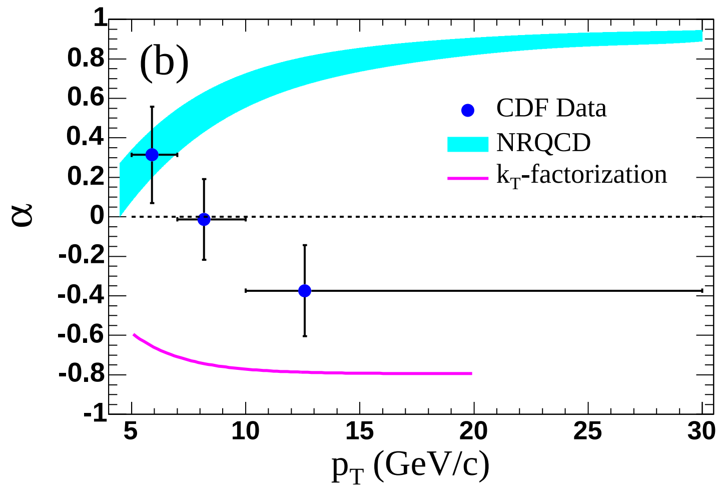
- $d\sigma/d(\cos\theta) \propto 1 + \alpha\cos^2\theta$.
  - $\alpha = 1$ is completely transverse;
  - $\alpha = -1$ is completely longitudinal.
- NRQCD prediction from Braaten, Kniehl, Lee (1999).
  - Feeddown from χ_c states is about 30% of the J/ψ sample and dilutes the polarization.
  - Feeddown from $\psi(2S)$ is about 10% of the J/ψ sample and is largely transversely polarized.
- Run I results are marginally compatible with the NRQCD prediction.
- Run II results are inconsistent with the NRQCD prediction.
- Also inconsistent with the Run I results. CDF was unable to track down the source of the Run I-Run II discrepancy.

$\psi(2S)$ Polarization in LO

Run: I



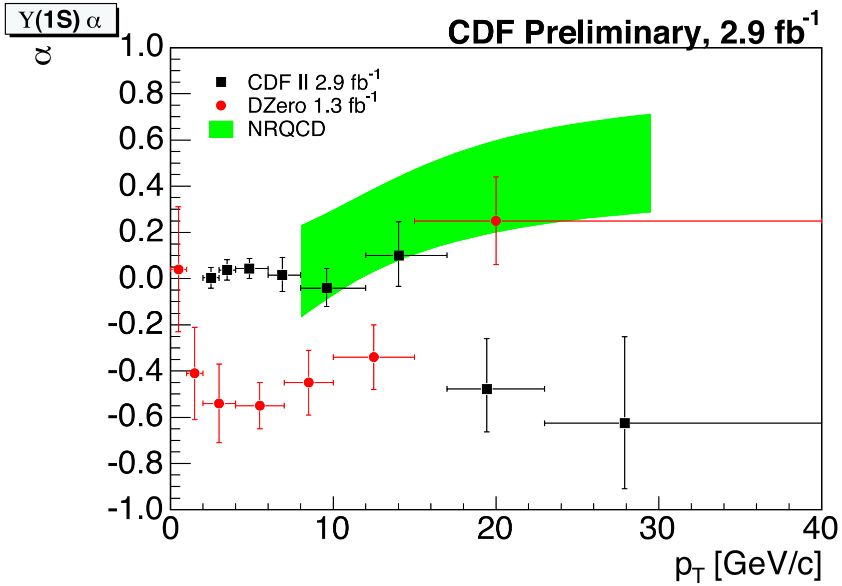
Run: II



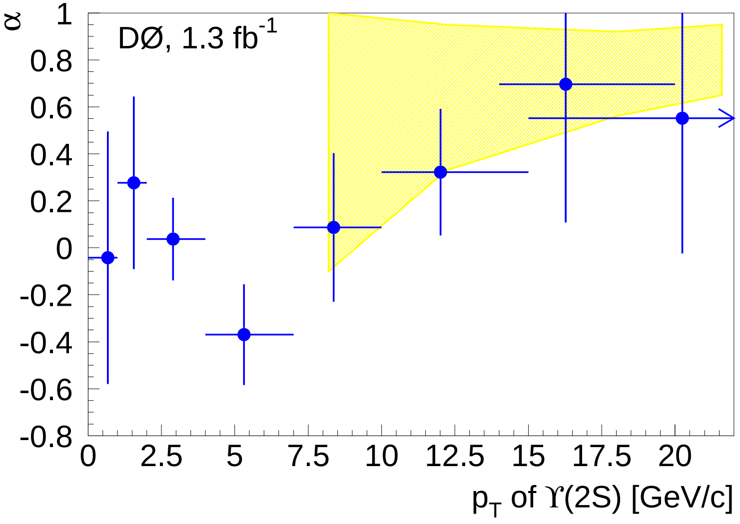
- The Run II data are incompatible with the LO NRQCD prediction.

Υ Polarization in LO

$\Upsilon(1S)$ Polarization:



$\Upsilon(2S)$ Polarization:



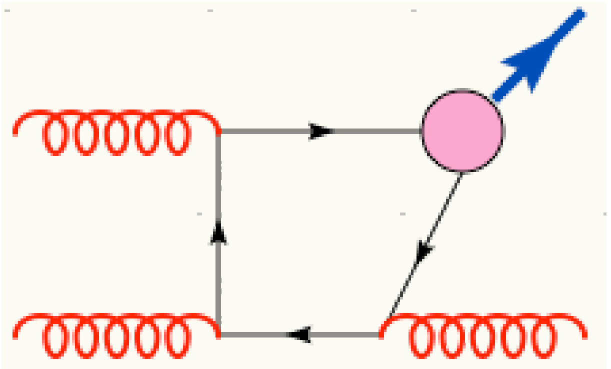
- In the $\Upsilon(1S)$ case, the D0 results (red) are incompatible with the CDF results (black).
- Both the CDF and D0 results are incompatible with the LO NRQCD prediction of Braaten and Lee (2000) (green), but in different regions of p_T.
- In the $\Upsilon(2S)$ case, the theoretical and experimental error bars are too large to make a stringent test.

Higher-Order Calculations

- Campbell, Maltoni, Tramontano(2007); Artoisenet, Lansberg, Maltoni (2007):
  Higher-order corrections to color-singlet quarkonium production at the Tevatron are unexpectedly large.

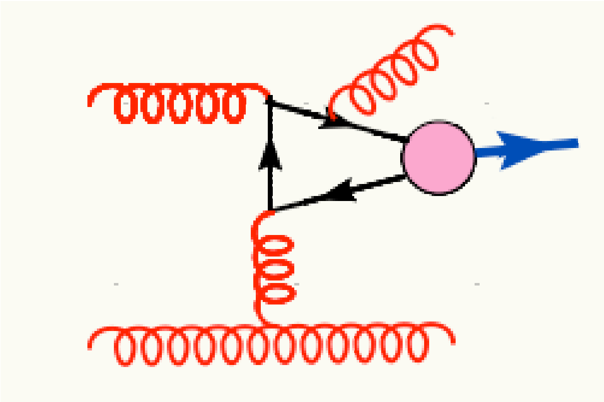
- At high p_T, higher powers of α_s can be offset by a less rapid fall-off with p_T.

LO:

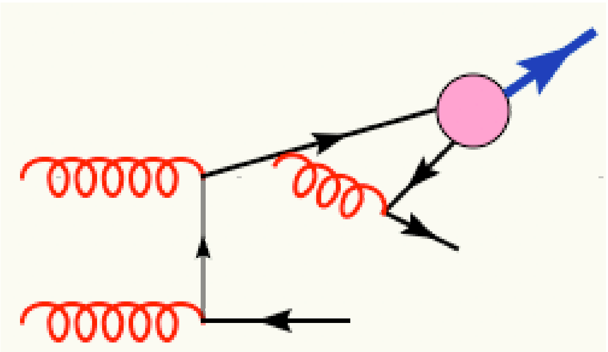


$$\sim \alpha_s^3 \frac{(2m_c)^4}{p_T^8}$$

NLO:

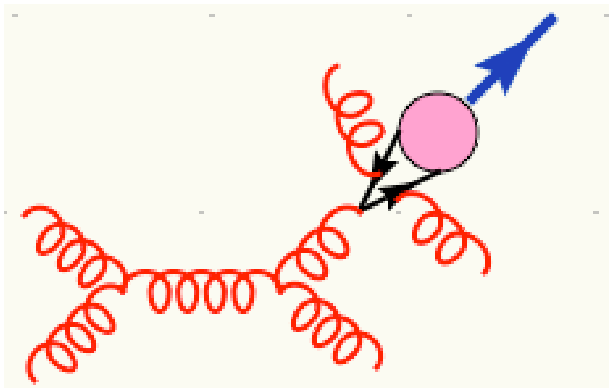


$$\sim \alpha_s^4 \frac{(2m_c)^2}{p_T^6}$$



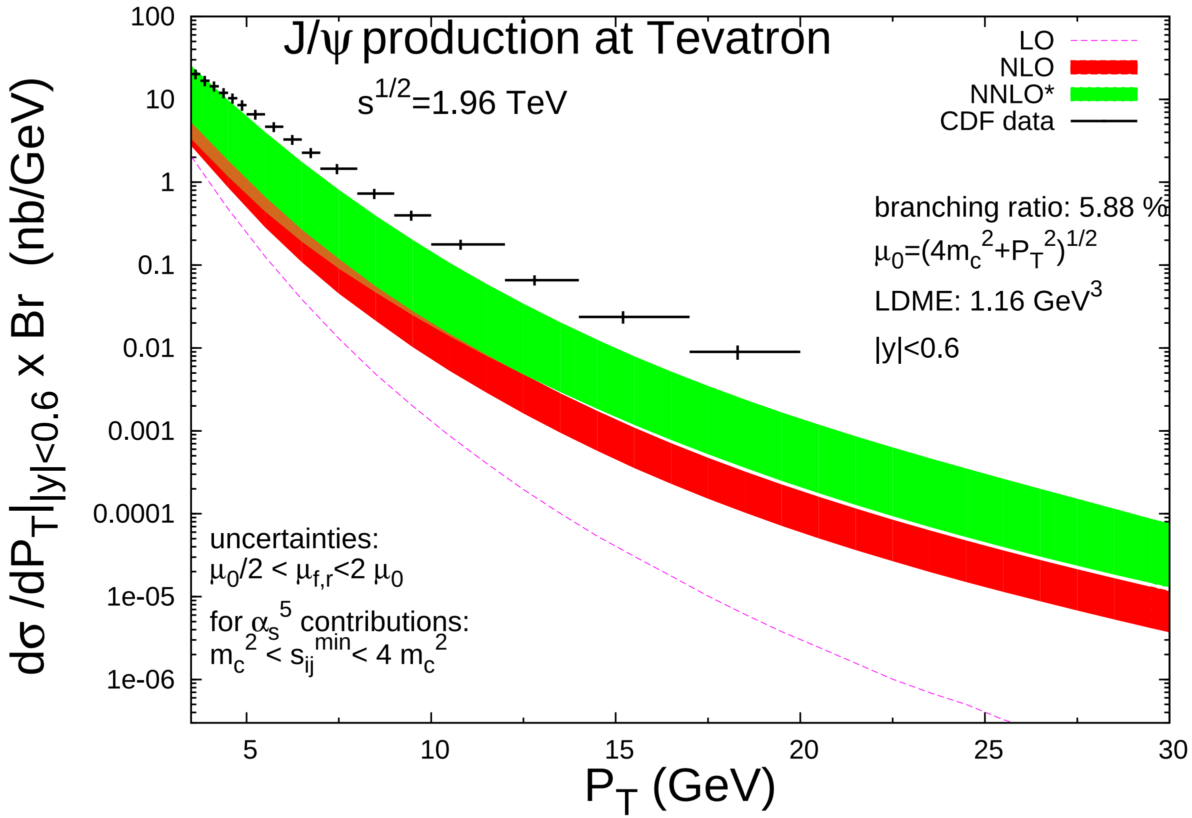
$$\sim \alpha_s^4 \frac{1}{p_T^4}$$

NNLO:



$$\sim \alpha_s^5 \frac{1}{p_T^4}$$

NLO and NNLO* Color-Singlet J/ψ Production



- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano.
- The NNLO* calculation is an estimate based on real-emission contributions only.
- The data still seem to require a color-octet contribution.

NLO Color-Octet S-Wave J/ψ and $\psi(2S)$ Production

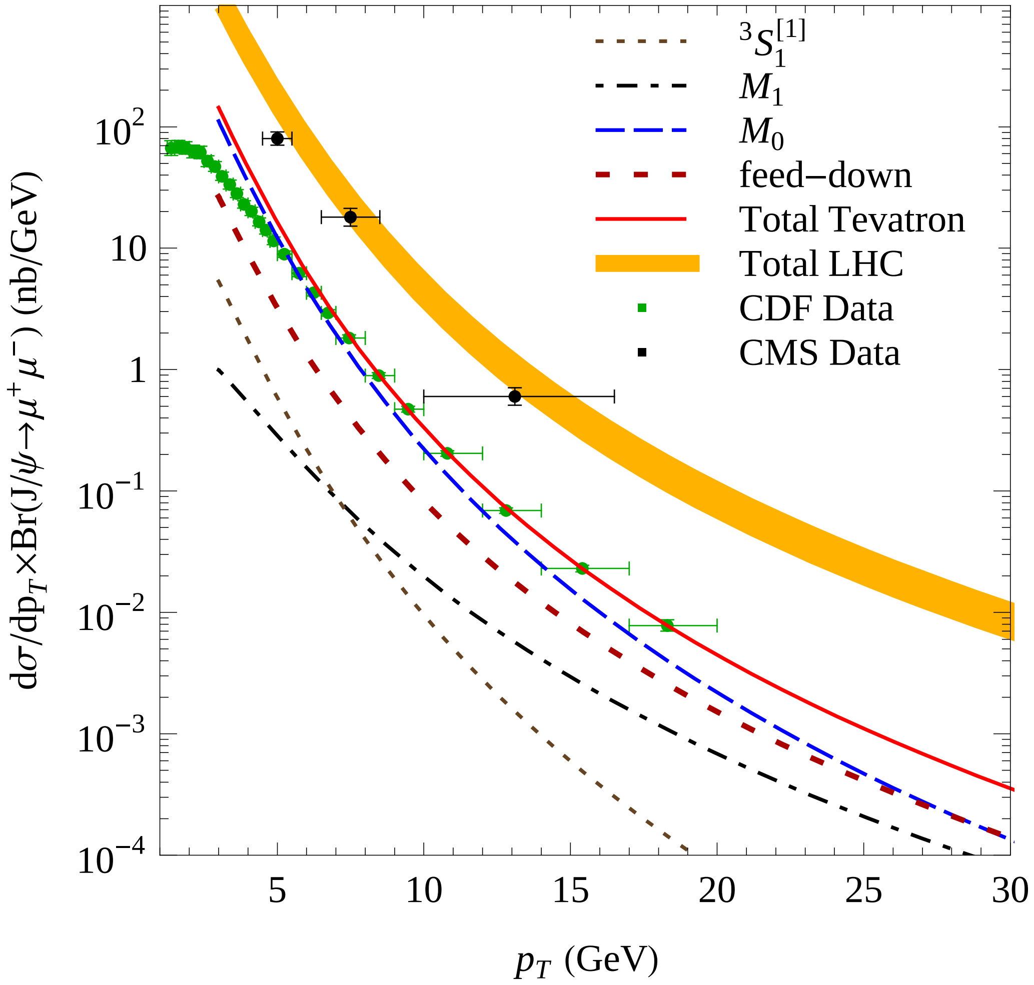
- Gong, Li, and Wang (2008, 2010): NLO corrections to the S-wave channels are small.
  - K factors at the Tevatron are about 1.235 for the ${}^1S_0$ channel and 1.139 for the ${}^3S_1$ channel.

First Complete NLO Color-Octet Calculations

Ma, Wang, and Chao (2010); Butenschön and Kniehl (2010)

- NLO corrections for all of the color-octet channels through order v^4.
  Color-octet channels: ${}^1S_0$, ${}^3S_1$, ${}^3P_J$.
- Confirm that the NLO corrections to the S-wave channels are small.
- Very large K factor ~ -10 for the ${}^3P_J$ channel.
  A $1/p_T^4$ contribution appears for the first time in NLO.
- The results of Ma, Wang, and Chao and Butenschön and Kniehl for the short-distance cross sections agree.

Ma, Wang, and Chao (2010):



- Matrix elements were fit to the CDF (2005, 2009) Run II data for $p_T > 7$ GeV.
- Feeddown from the $\psi(2s)$ was taken into account by using the CDF (2005, 2009) Run II data.
- Feeddown from the χ_{cJ} states was taken into account by using the NLO prediction of Ma, Wang, and Chao (2010) for χ_{cJ} production.
  - Uses a color-octet matrix element that is obtained by fitting to the CDF (2007) measurements of $R_{\chi c} = \sigma_{\chi_{c2}}/\sigma_{\chi_{c1}}$.
  - The predicted χ_{cJ} fraction increases with increasing p_T, while the χ_{cJ} fraction measured by CDF (1997) in Run I decreases with increasing p_T.
- The fits were used to predict the CMS (2010) data.

- Only the linear combinations

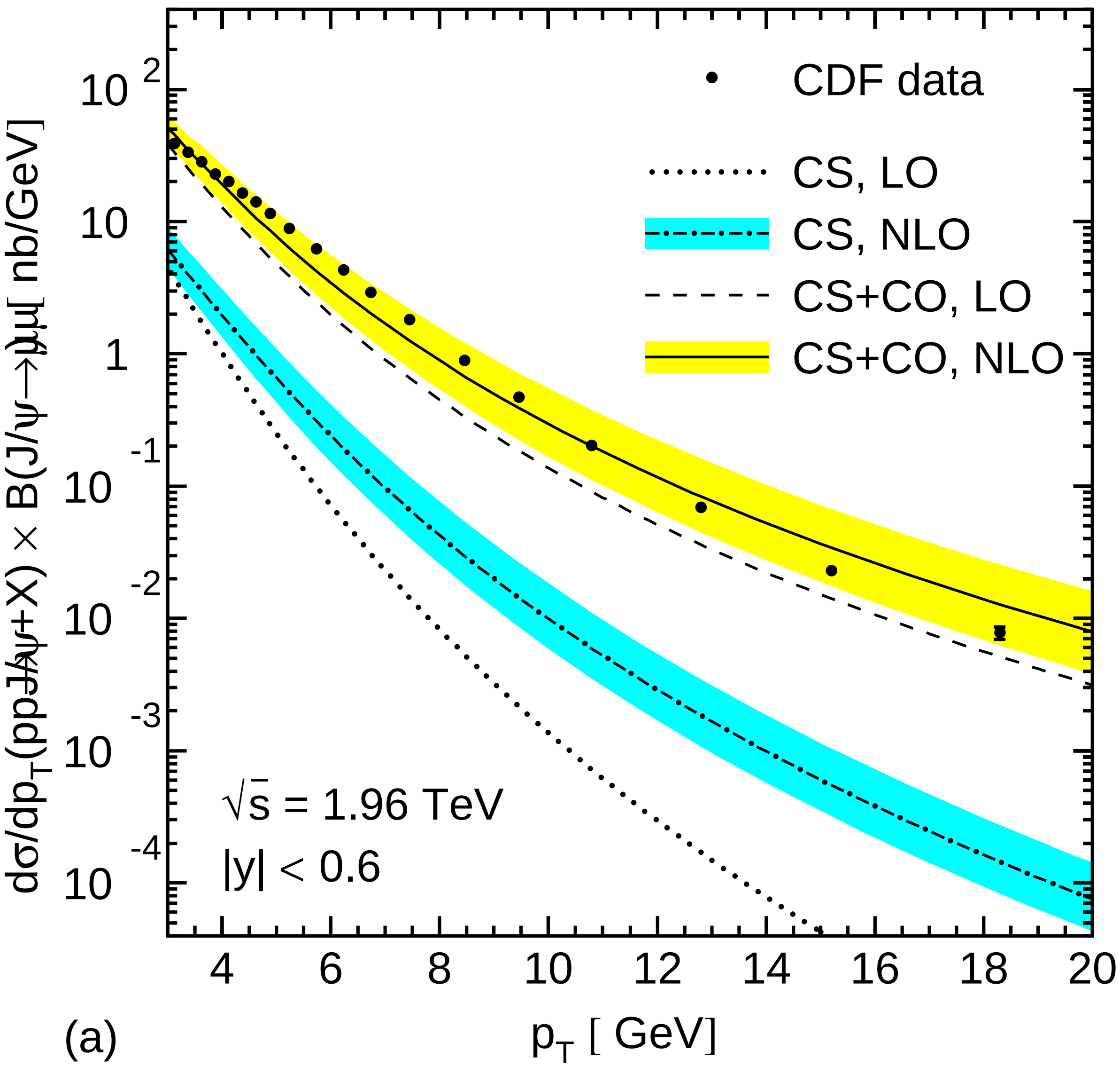
$$M_{0,r_0} = \langle O^\psi({}^1S_0^{[8]})\rangle + (r_0/m_c^2)\langle O^\psi({}^3P_0^{[8]})\rangle = (7.4 \pm 1.9) \times 10^{-2}\ \text{GeV}^3$$
$$M_{1,r_1} = \langle O^\psi({}^3S_1^{[8]})\rangle + (r_1/m_c^2)\langle O^\psi({}^3P_0^{[8]})\rangle = (0.05 \pm 0.02) \times 10^{-2}\ \text{GeV}^3$$

could be fit unambiguously.

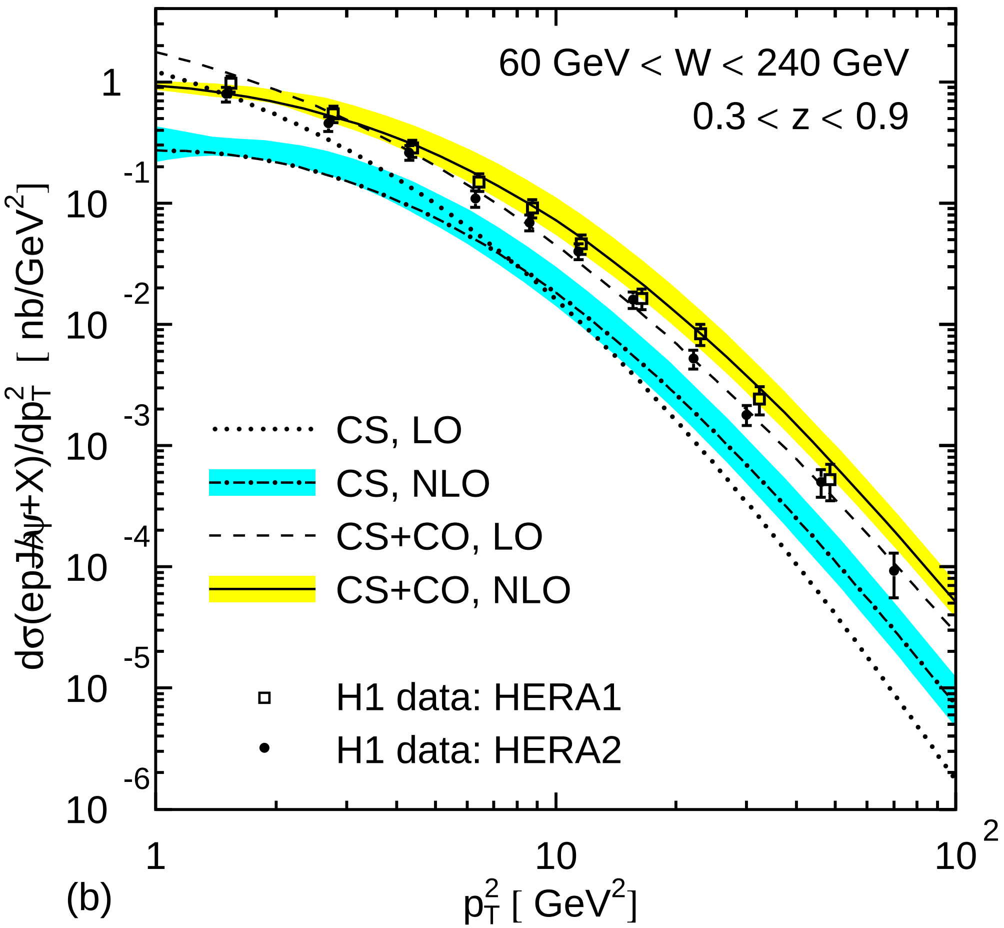
$r_0 = 3.9$ and $r_1 = -0.56$ chosen on the basis of approximate relations between the short-distance coefficients.

- The small size of M_{1,r_1} suggests that $\langle O^\psi({}^3S_1^{[8]})\rangle$ is small.
  - Assumes that there is not an accidental cancellation between the $\langle O^\psi({}^3S_1^{[8]})\rangle$ and $\langle O^\psi({}^3P_0^{[8]})\rangle$.
  - Might explain the absence of transverse J/ψ polarization in the Tevatron data.

Butenschön and Kniehl (2010):



- NRQCD matrix elements were extracted in a fit that made use of both the CDF (2005) Run II data and the H1 (2002, 2005) HERA I and HERA II data.
- All three color-octet NRQCD matrix elements were determined in the fit.
- A cut $p_T > 3$ was applied to the CDF data.
- No corrections were made for feed-down.
- This fit describes shape of the CDF data less well than the fit of Ma, Wang, and Chao.
  - May be caused by tension between the theory and the combined CDF and H1 data.
- The results were used to predict cross sections at PHENIX and CMS.

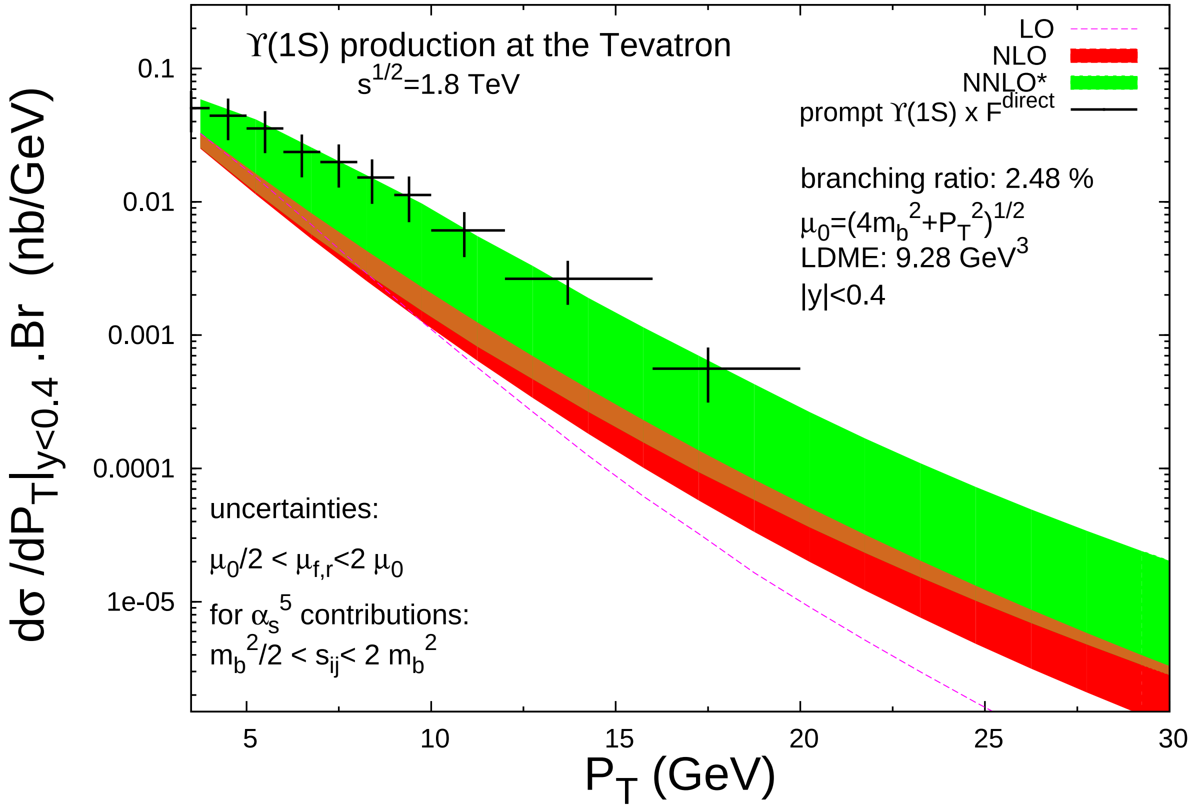


There is a slight discrepancy in shape between the NLO prediction and the H1 data.

Discussion

- The Butenschön and Kniehl matrix elements are not very different from those from LO extractions.
- In comparison to the values in the Ma, Wang, and Chao fit
  - M_{0,r_0} is about a factor 4 smaller,
  - M_{1,r_1} is about a factor 11 larger.
- Since the short-distance cross section agree, the differences between the matrix elements must arise from the differences in the fitting procedures.
- The differences in the matrix-element extractions seem to arise mainly from
  - The use of the HERA data in the fit of Butenschön and Kniehl.
    Note that most of it is at rather low values of p_T.
  - The use of approximate relations between the short-distance coefficients to select the linear combinations that are used in the fit of Ma, Wang, and Chao.
  - The inclusion of feeddown from the $\psi(2S)$ and χ_{cJ} states in the fit of Ma, Wang, and Chao.
    The calculated χ_{cJ} feeddown may fall less rapidly with p_T than the CDF data.
- The relative size of the $\langle O^\psi({}^3S_1^{[8]})\rangle$ contribution and the expected J/ψ polarization depend on the resolution of these discrepancies.

NLO and NNLO* Color-Singlet Υ Production



- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008)
- NLO results confirmed by Gong and Wang (2007).

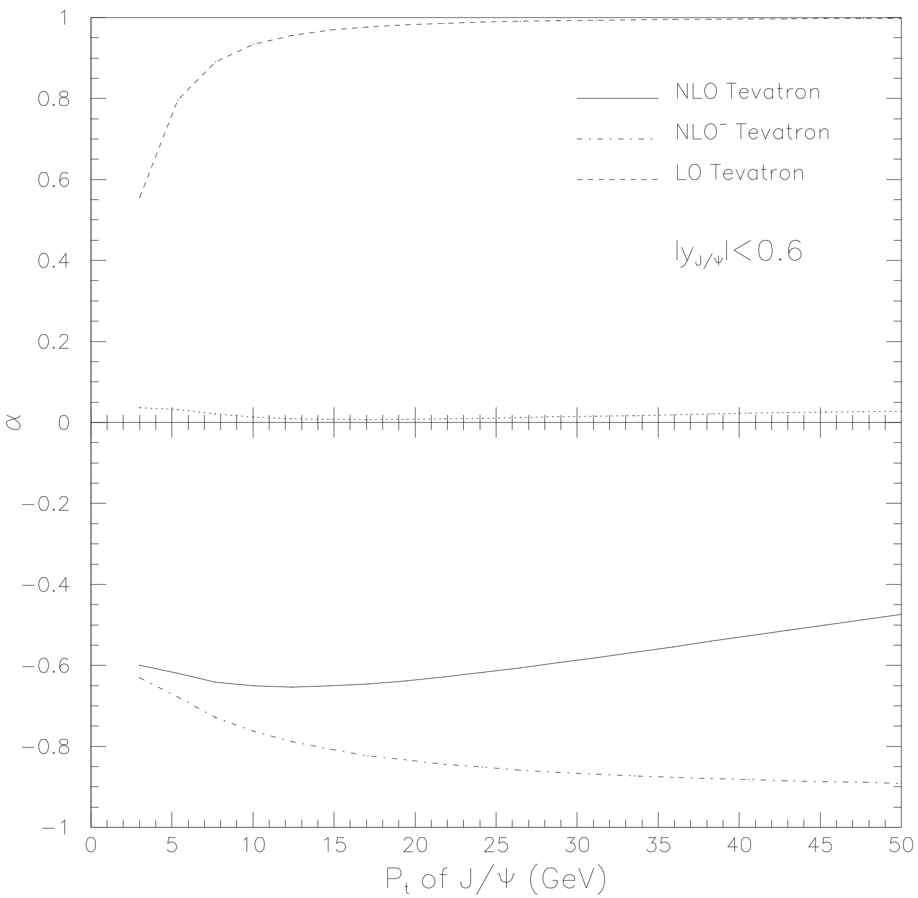
- The data could be explained by color-singlet production alone.
- There is still room for a substantial amount of color-octet production.
- Color-octet production is suppressed as v^4.
  Should be smaller for Υ $(v^2 \approx 0.1)$ than for J/ψ $(v^2 \approx 0.3)$.

NLO Color-Octet S-Wave Υ Production

- (Gong, Wang, Zhang (2008, 2010)): NLO corrections to the S-wave channels are small.
- K factors at the Tevatron are about 1.313 for the ${}^1S_0$ channel and 1.379 for the ${}^3S_1$ channel.

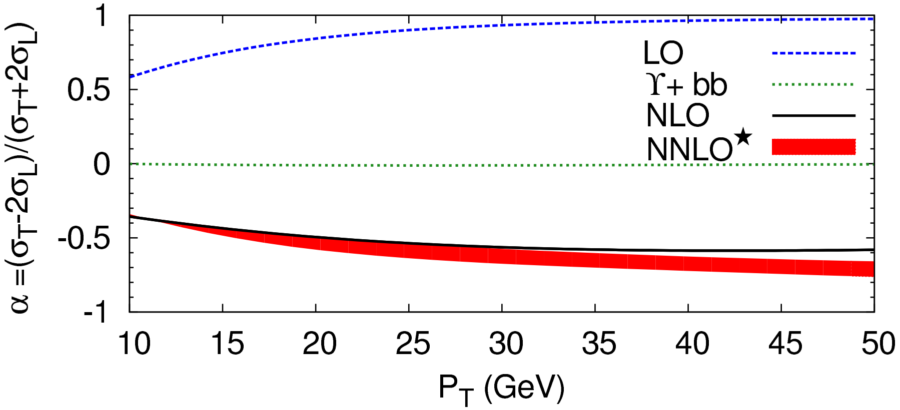
NLO and NNLO* Color-Singlet Polarization

- Gong and Wang (2008): color-singlet J/ψ polarization at the Tevatron changes from transverse to longitudinal when NLO corrections are included.



- NLO$^-$ excludes $gg \to J/\psi c\bar{c}$.
- Unlabeled line is contribution of $gg \to J/\psi c\bar{c}$.

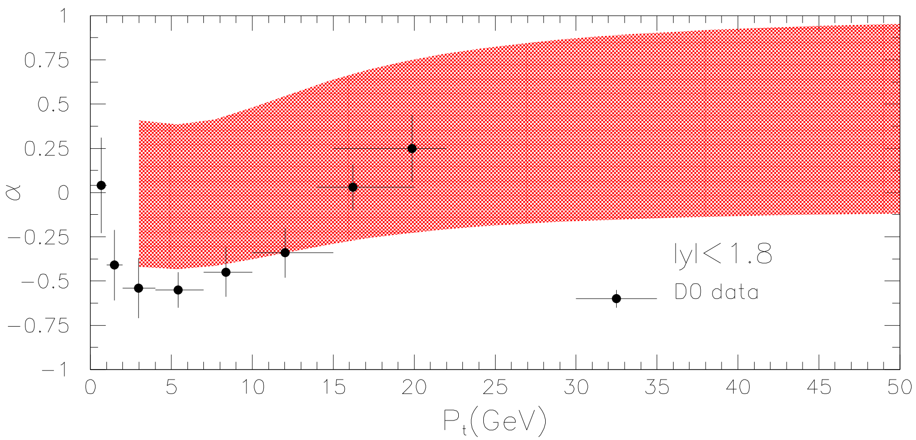
- Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008): color-singlet Υ polarization at the Tevatron changes from transverse to longitudinal when NLO and NNLO* corrections are included.



- NLO result confirmed by Gong and Wang (2008).

NLO Color-Octet S-Wave Polarization

- Gong, Li, and Wang (2008): The prediction for the J/ψ polarization is little affected by NLO corrections to the color-octet ${}^1S_0$ and ${}^3S_1$ channels.
- Gong, Wang, and Zhang (2010): The prediction for the Υ polarization is not shifted significantly by NLO corrections to the color-octet ${}^1S_0$ and ${}^3S_1$ channels.



- There are large uncertainties because of the feeddown from the χ_{bJ} states.

Fragmentation-Function Approach to Quarkonium Production

- Large corrections appear in NLO and NNLO* because new channels that open produce a slower fall-off with increasing p_T.
  - The new channels spoil the convergence of the perturbation series.
  - There are still large renormalization-scale uncertainties in NLO and NNLO*.
- The fragmentation approach of Kang, Qiu, and Sterman (2010) potentially brings the higher-order corrections under better control.
- Re-organizes the perturbation expansion according to powers of p_T.
- In the fragmentation functions, an important class of higher-order corrections is resummed by making use of evolution equations for the fragmentation functions.
- It may be possible to compute fragmentation contributions to higher orders in α_s than one can compute complete cross sections.

Discussion

- The NNLO* corrections greatly increase the color-singlet contributions to the J/ψ and Υ cross sections, but the uncertainties are very large.
- The J/ψ production data still seem to require a color-octet contribution that dominates at large p_T.
- A color-octet contribution is not required or excluded by the Υ production data.
- NLO corrections might change our ideas about the relative contributions of the color-octet channels and about the expected quarkonium polarization.
- The fragmentation approach may help to reduce theoretical uncertainties.
- Interpretation of the Tevatron J/ψ data, both polarized and unpolarized, is complicated by feed-down from the $\psi(2S)$ and χ_{cJ} states.
- High-statistics, high-p_T measurements of the cross section and polarization for direct production of the J/ψ, χ_{cJ}, and $\psi(2S)$ states would be of great help.
- The discrepancies between the CDF and D0 Υ polarization data must be resolved before any meaningful comparisons can be made with theory.
- NLO calculations of the color-octet P-wave contributions to quarkonium polarization are needed.

χ_{cJ} Production

- Ratio of P-wave cross sections:

$$R_{\chi_c} = \frac{d\sigma_{\chi_{c2}}/dp_T}{d\sigma_{\chi_{c1}}/dp_T}.$$

- In NRQCD factorization in LO, R_{χ_c} is dominated by color-octet contributions at large p_T. It is predicted at large p_T to be

$$R_{\chi_c} = 5/3.$$

- CDF (2007): At large p_T

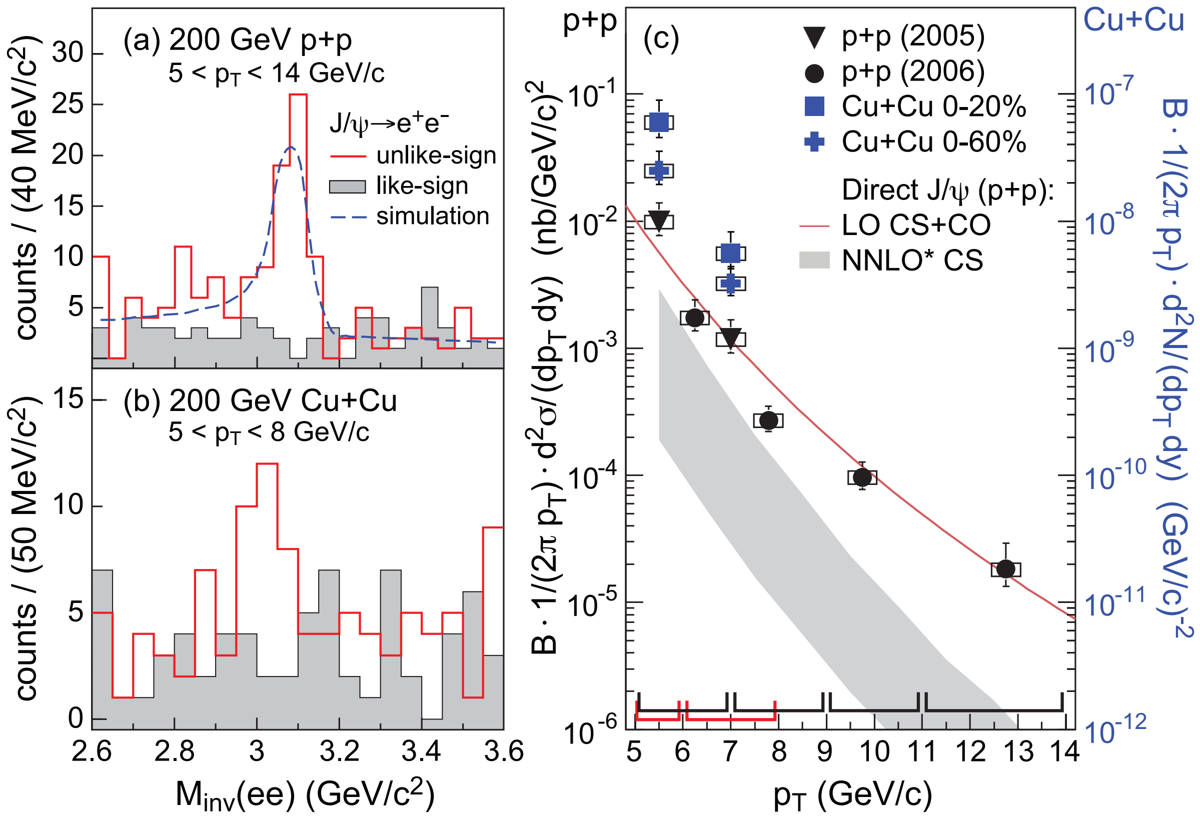
$$R_{\chi_c} \approx 0.75.$$

- Ma, Wang, Chao (2010): NLO corrections to R_{χ_c} are large at large p_T.
  - Using the NLO results, they are able to fit the p_T distribution of R_{χ_c}, using plausible values of the color-octet NRQCD matrix elements.
  - The fit predicts that feeddown from the χ_{cJ} states to the J/ψ, may be as large as 30% of the J/ψ rate at $p_T = 20$ GeV.
  - The predicted χ_{cJ} fraction increases with increasing p_T, while the χ_{cJ} fraction measured by CDF (1997) in Run I decreases with increasing p_T.
    The experimental and theoretical uncertainties are large.

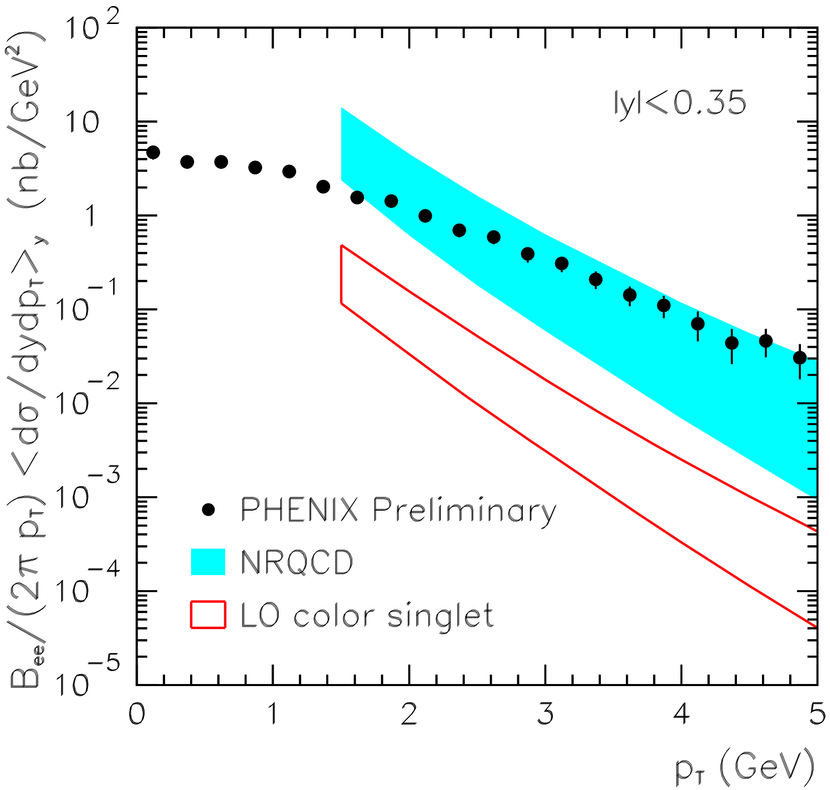
J/ψ Production at RHIC

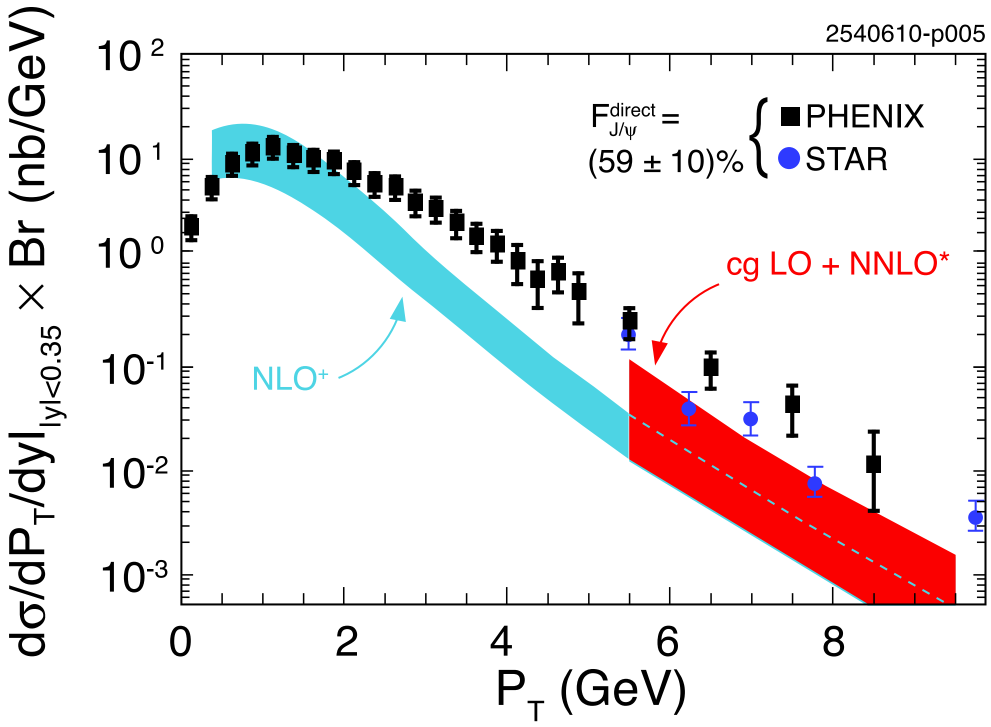
Production Cross Section

- The STAR collaboration has measured the J/ψ p_T distributions in $p + p$ and Cu+Cu collisions:



- Nayak, Liu, Cooper (2003): An LO NRQCD calculation (color-singlet plus color-octet contributions) fits the data well.
  - Does not include feeddown from $\psi(2S)$, χ_c, or B decays. (Estimated to be a factor 1.5.)

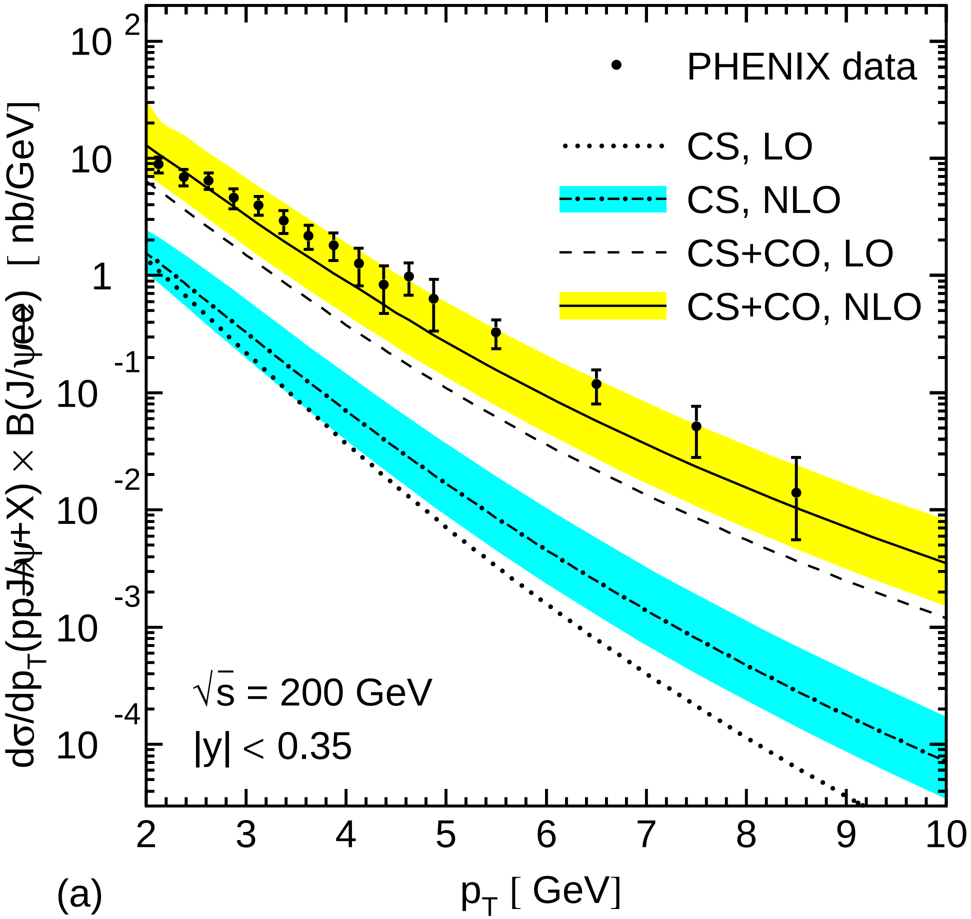




- Chung, Yu, Kim, Lee (2010): An LO NRQCD calculation, including feeddown, fits the PHENIX (2009) data well.
- The color-singlet contribution is well below the PHENIX (2009) data.

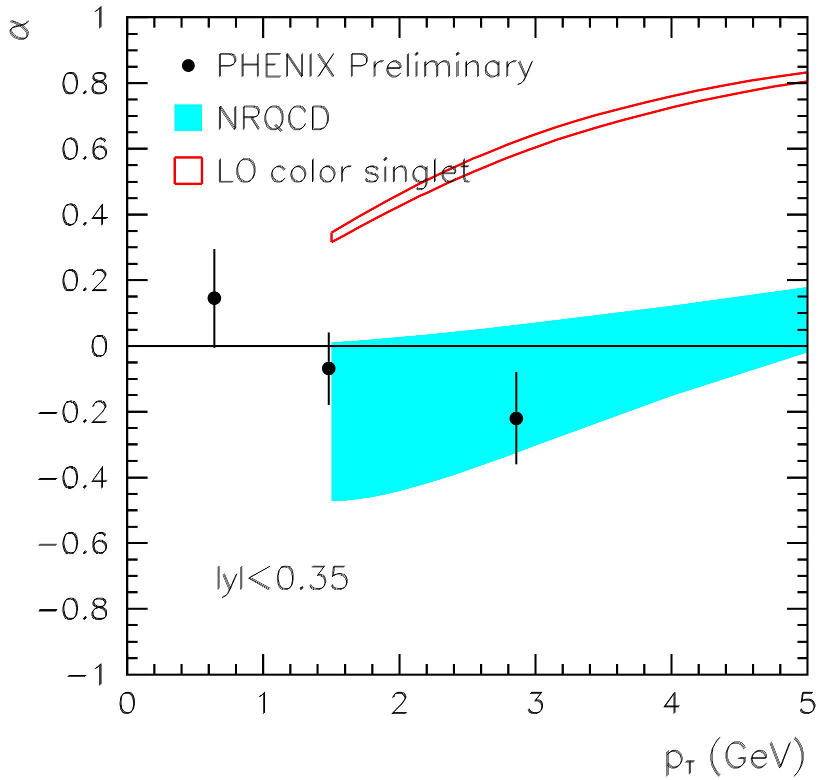
- Lansberg (2010): NLO corrections increase the size of the color-singlet contribution substantially.
- The color-singlet contribution still lies below the PHENIX (2010) and STAR (2009) data at large p_T.
- Figure courtesy of Hee Sok Chung.

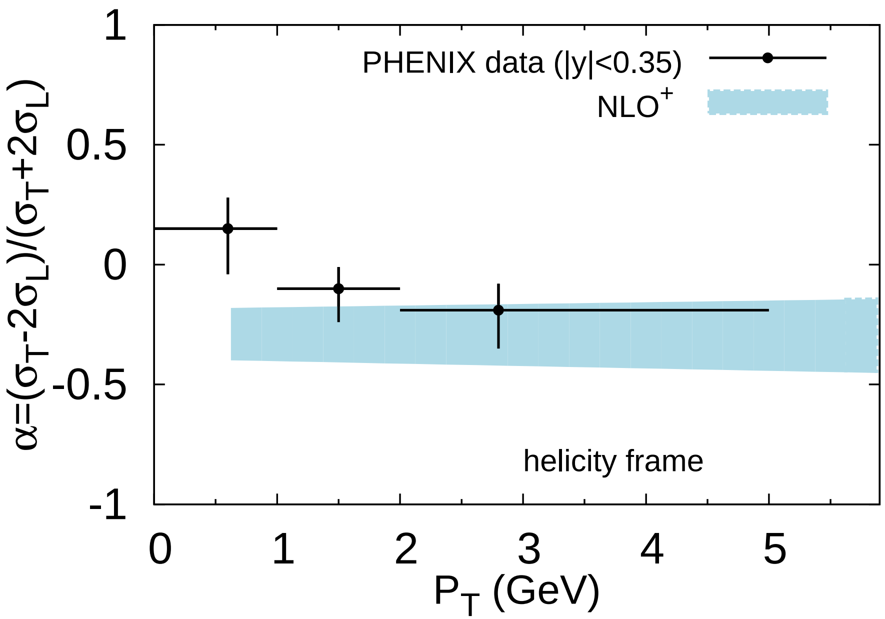
- The NLO NRQCD calculation of Kniehl and Butenschön (2010), with NRQCD matrix elements fit to the CDF (2005) and H1 (2002, 2010) data, agrees well with the PHENIX data:



- Feeddown ($\approx 36\%$) is not included in the theoretical prediction.
- The NLO color-singlet contribution is well below the PHENIX data.

Polarization

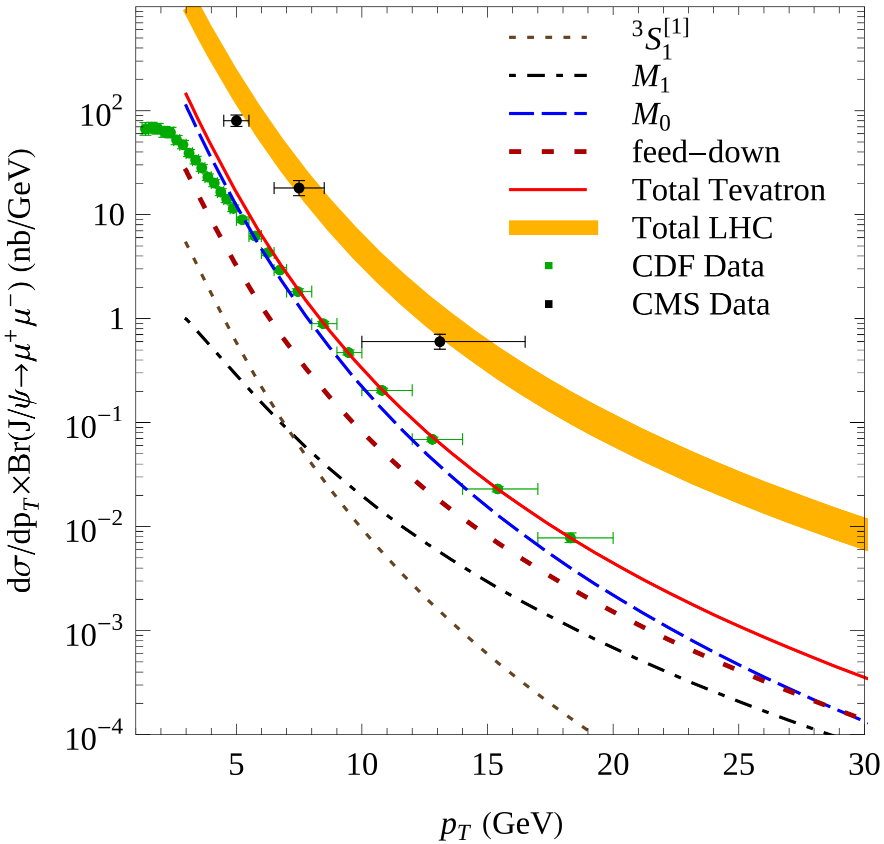




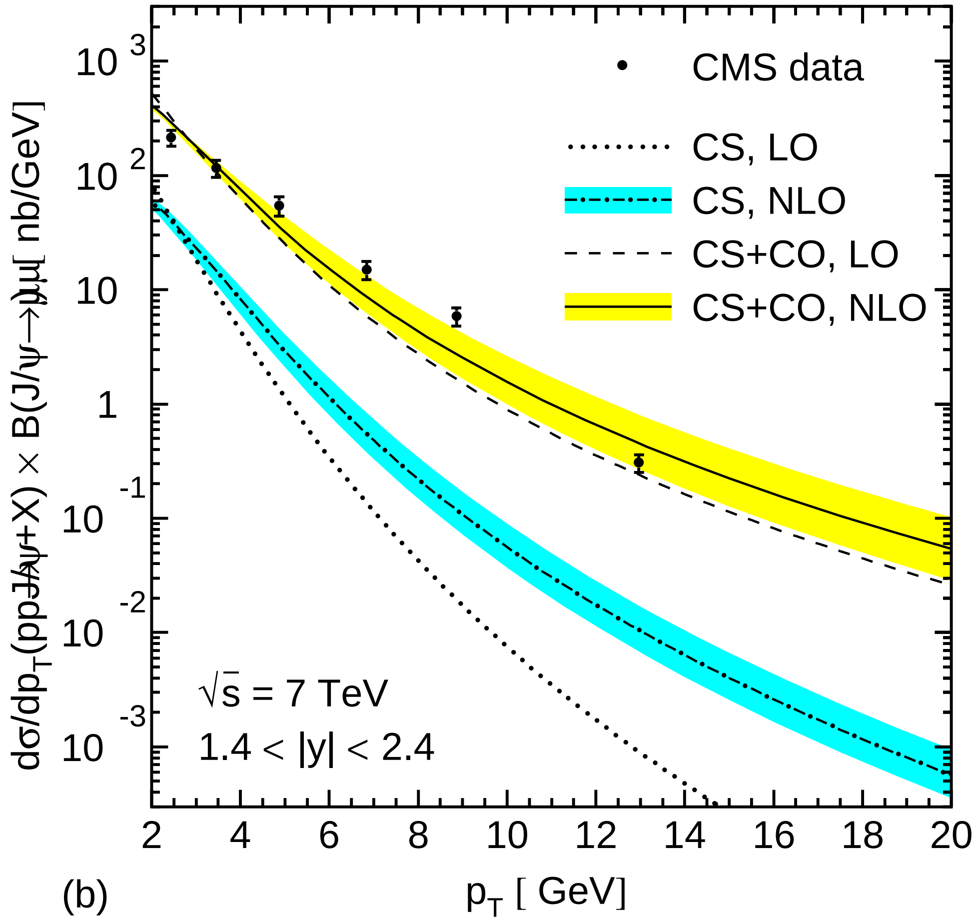
- Chung, Yu, Kim, Lee (2010): An LO NRQCD calculation, including feeddown, fits the PHENIX (2009) data well.
- The color-singlet contribution in LO is in poor agreement with the PHENIX data.
- Lansberg (2010): The NLO corrections to the color-singlet contribution make it virtually indistinguishable from the color-octet contribution.

J/ψ Production at the LHC

- The NLO predictions of Ma, Wang and Chao (2010) and Kniehl and Butenschön (2010) agree well with the CMS (2010) data:



Ma, Wang, and Chao (2010)
$|y| < 2.4$

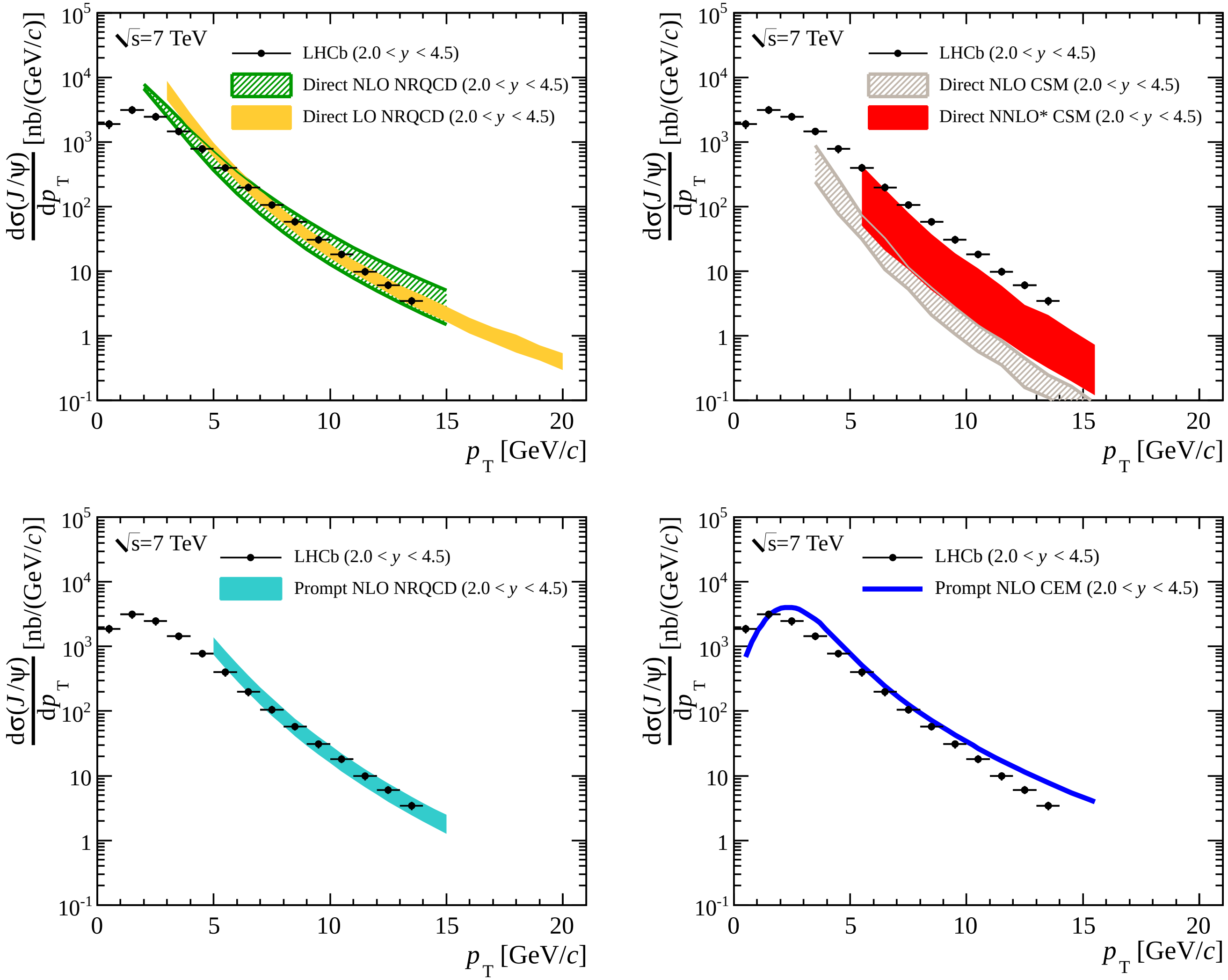


Butenschön and Kniehl (2010)
$1.4 < |y| < 2.4$

- Only the calculation of Ma, Wang, and Chao (2010) includes the effects of feeddown.

- Somewhat surprising that both calculations agree well with the data since the NRQCD matrix elements that are used are so different.
  - The J/ψ cross sections at the Tevatron and the LHC are dominated by gg-initiated processes.
  - The fit to the Tevatron p_T distribution produces a mapping of the gluon momentum values into a p_T spectrum.
  - Because the p_T distributions of the three important color-octet channels are not linearly independent, that mapping can be achieved in NRQCD in different ways.
  - Two models that have the same mapping of gluon momenta to p_T distributions will produce the same predictions for $d\sigma/dp_T$ at the Tevatron and the LHC.
  - Explains why both the LO and NLO NRQCD predictions fit the Tevatron data and predict the LHC data accurately.
- The analysis of Kang, Qiu, and Sterman shows that we can predict only the leading and first subleading powers of p_T^2 in $d\sigma/dp_T$.
- Clearly, we need additional observables in order to understand the details of the production mechanism.

- The LHCb data also agree well with the NLO NRQCD predictions.
  (There are also Atlas and Alice measurements.)



Top left: Butenschön and Kniehl (2010). Top right: Artoisenet *et al.* (2008), Lansberg (2009).
Bottom left: Ma, Wang, and Chao (2010). Bottom right: Frawley, Ullrich, and Vogt (2008).

- The NNLO* color-singlet prediction significantly undershoots the data.

Summary

- The NRQCD factorization approach provides a systematic method for calculating quarkonium decay and production rates as double expansions in powers of α_s and v.
- NRQCD factorization for inclusive production rates has not yet been established.
- NRQCD factorization has enjoyed a number of successes:
  - quarkonium production at the Tevatron,
  - J/ψ production at RHIC,
  - J/ψ production at the LHC,
  - $\gamma\gamma \to J/\psi + X$ at LEP,
  - inelastic J/ψ photoproduction at HERA,
  - J/ψ production in DIS at HERA,
  - exclusive double-charmonium production at Belle and BaBar.

- The disagreement between theory and experiment for quarkonium polarization at the Tevatron presents a serious challenge.
  - The CDF results for the J/ψ and $\psi(2S)$ polarizations are unconfirmed.
  - The CDF and D0 results for the Υ polarization do not agree.
  - NLO calculations of the P-wave contributions to quarkonium polarization are needed in order to draw definite conclusions.
- In a number of cases, corrections of higher order in α_s and v and resummations near kinematic endpoints have proven to be essential to obtain reliable theoretical predictions.
- NNLO* calculations of color-singlet quarkonium production at the Tevatron may reduce the importance of color-octet contributions and could possibly resolve some puzzles
- In many cases, the perturbation expansion converges poorly, and theoretical uncertainties are large.
- The fragmentation approach of Kang, Qiu, and Sterman may help to bring theoretical uncertainties under control.
- Measurements of direct-production cross sections and polarizations would be of great help in understanding production mechanisms.
- We need to make additional measurements beyond $d\sigma/dp_T$ at hadron-hadron colliders in order to pin down the quarkonium production mechanisms.